Mid Semester Examination

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February 22^{nd}, 2024 (Morning Session)
Duration: 180 minutes.
Total points: 80

Please give arguments where necessary. If it is unclear from your answer why a particular step is being taken, full credit will not be awarded. Grades will be awarded not only based on what final answer you get, but also on the intermediate steps.

1. In a spherically symmetric configuration, the region $r < b$ has a uniform charge density ρ_b. The region $b < r < a$ contains a uniform charge density ρ_a. Ar $r = b$ there is no surface charge, while at $r = a$ the surface charge density is chosen in such a way that $\vec{E} \equiv 0$ for $r > a$.

   (a) Determine $\vec{E}$ everywhere.

   (b) Determine the surface charge density at $r = a$.

   (c) Now consider a situation such that a uniform surface charge density σ_0 is added to the surface at $r = b$. Carry out the same calculations as in the previous two parts, still under the condition that $\vec{E} = 0$ for $r > a$.

   5 + 4 + (6 + 5) = 20 points

2. There are two infinite sheets of uniform surface charge density, both perpendicular to the z-axis; the one at $z = s/2$ carries a surface charge density σ_0, while the one at $z = -s/2$ contains $-\sigma_0$. The space between is filled with a volume charge density that varies with position as $\rho = 2\rho_0 \frac{z}{s}$. The electric field for $z < -s/2$ sheet is uniform, given by $\vec{E} = E_0 \hat{e}_z$. Find the electric field everywhere.
   10 points

3. Consider a sphere with radius R with the following charge density on its surface:
   $$\sigma(r, \theta, \phi) = \sigma_0 \sin^2\theta \sin^2\phi$$
   Find the potential $\varphi(r, \theta, \phi)$ everywhere. (Hint: Think about trying an expansion in the proper co-ordinate system).
   15 points

4. A short piece of wire of length L carrying charge Q, is placed along the z-axis, centred at the origin. The detailed charge distribution $\lambda(z)$ of the wire is not known, except the charge density is an even function of z. A new length-scale $\mathcal{L}_0$ can be defined in the problem, which captures the rms length of the charge distribution:

$$Q\mathcal{L}_0^2 = \int_{\textbf{wire}} dz z^2 \lambda(z) \tag{1}$$

   (a) Find the potential everywhere upto and including quadrupole order.

   (b) If, instread, $\lambda(z)$ was odd in z, what would the potential be upto and including quadrupole order?

   7 + 8 = 15 points

5. (a) In a spherical region surrounding the origin, the charge density is spatially uniform but has a time-dependence $\rho(\vec{r}, t) = \rho_0(t)$, with $\dot{\rho_0}(t) \neq 0$. Find the current density everywhere, assuming spherical symmetry.

   (b) In the region $z > 0$, the current density $\vec{J} \equiv 0$. In the region $z < 0$, the current density is given by $\vec{J} = J_0(x, y) \cos(\omega t)\, \hat{e}_z$. At $t = 0$, $\sigma(x, y, z = 0) = 0$. Find $\sigma\,(x, y, 0, t > 0)$.

   4 + 6 = 10 points

6. Consider a conducting spherical shell of radius R and thickness t, $t \ll R$. Through a tiny hole in the surface (which has negligible influence on the electrostatics), a charge q is moved quasi-statically to the center of the shell from spatial infinity. Find the work done during the process.
   10 points